Before Calculus AB Study Guide

✨ **Welcome to Calculus!** ✨

Calculus is the math of motion and change.
It helps us understand how things grow, move, and react—whether it's the curve of a rollercoaster, the speed of a rocket, or the rate your coffee cools down! ☕ 🚀 📈

Over the Summer, you should work this review packet to prepare for a Calculus review assessment on Monday August 18th! Work diligently on these review questions and be sure to ask any questions you might have when we return.

With love,

Ms. Mayers

Part A: Factor Completely

1. $x^2 + 5x + 6$

2. $x^2 - 9$

3. $2x^2 + 7x + 3$

4. $x^2 - 16x + 64$

5. $3x^2 - 12x$

6. $4x^2 - 25$

Part B: Factor by Grouping

1. $x^3 + 3x^2 + 2x + 6$

2. $2x^3 + 5x^2 + 4x + 10$

Part C: Special Factoring Patterns

1. $x^2 - 36$

2. $x^2 + 6x + 9$

3. $x^3 + 8$

4. $27x^3 - 1$

Part D: Challenge Problems

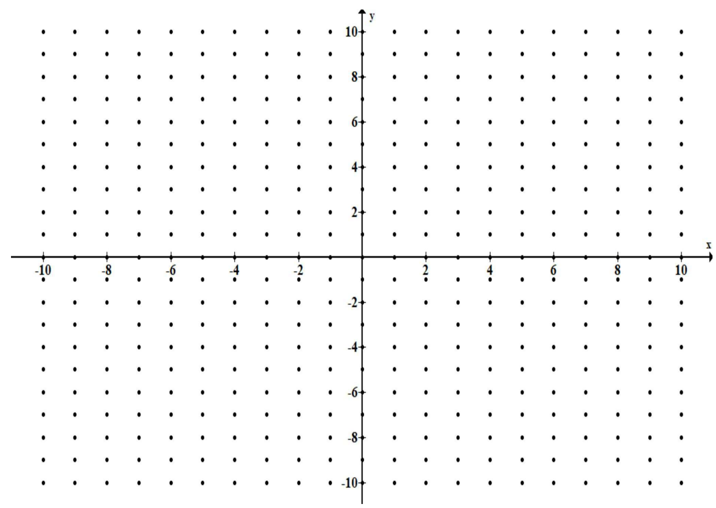
1. $x^4 - 81$

2. $x^3 - 6x^2 + 11x - 6$

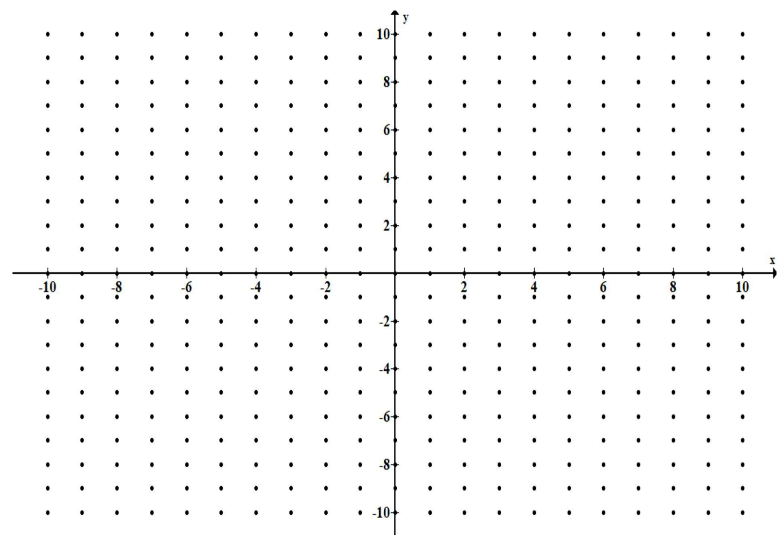
3. $x^2(4x^2 - 9)$

Graph each piecewise function.

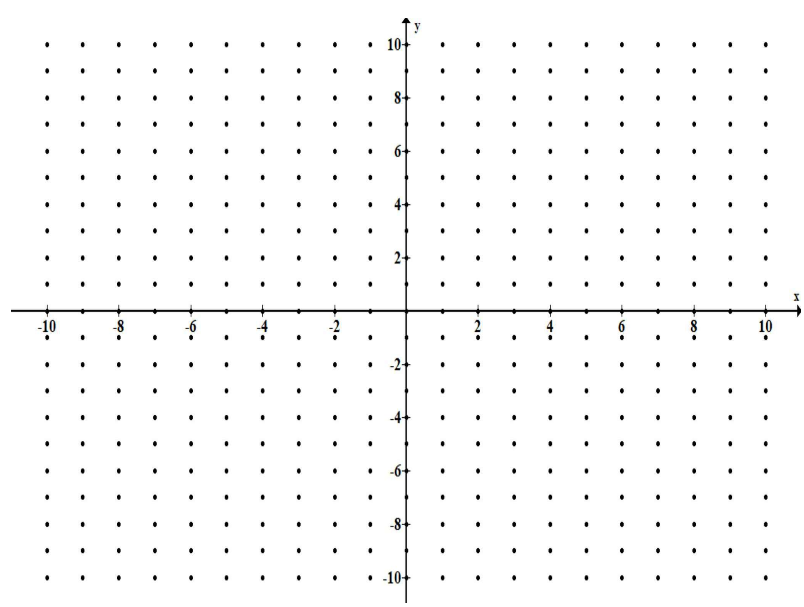
36. $f(x) = \begin{cases} x + 3 & ; \ x < 0 \\ -2x + 5 & ; \ x \geq 0 \end{cases}$



37. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x - 3 & ; \ x > 2 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2 - |x - 2| & ; \ x > 1 \end{cases}$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.

39. $5^x = \frac{1}{5}$ ____________

40. $6^x = 1296$ ____________

41. $6^{2x-7} = 216$ ____________

42. $5^{3x-1} = 49$ ____________

43. $10^{x+5} = 125$ ____________

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$ ____________

45. $e^{2\ln 3} =$ ____________

46. $\ln e^9 =$ ____________

47. $5\ln e^3 =$ ____________

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$ ________________

49. $3e^x = 120$ ________________

50. $e^x - 8 = 51$ ________________

51. $\ln x = 2.5$ ________________

52. $\ln(3x - 2) = 2.8$ ________________

53. $2\ln(e^x) = 5$ ________________

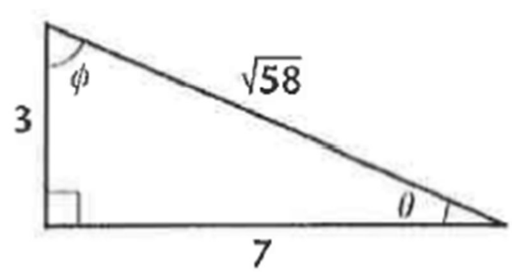
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do <u>NOT</u> need to rationalize the denominator.

67. $\sin\theta =$ _________

68. $\cos\theta =$ _________

69. $\tan\phi =$ _________

70. $\sec\phi =$ _________



Double-Angle Formulas & Unit Circle Practice

◆ Part A: Unit Circle Evaluation

Evaluate the following using the unit circle. Give **exact values** (no decimals).

1. $\sin(\pi/6)$
2. $\cos(\pi/3)$
3. $\tan(\pi/4)$
4. $\sin(3\pi/4)$
5. $\cos(5\pi/6)$
6. $\tan(7\pi/6)$
7. $\sin(11\pi/6)$
8. $\cos(3\pi/2)$

Inverse Trig Functions & The Unit Circle

Part A: Evaluate the Following Without a Calculator

Use the unit circle to find exact values. Give answers in radians.

1. $\sin^{-1}\left(\frac{1}{2}\right) =$
2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$
3. $\tan^{-1}(1) =$
4. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$
5. $\cos^{-1}(0) =$
6. $\tan^{-1}(\sqrt{3}) =$
7. $\cos^{-1}(-1) =$
8. $\sin^{-1}(0) =$
9. $\tan^{-1}(0) =$
10. $\sin^{-1}\left(-\frac{1}{2}\right) =$

🔹 Double-Angle Formula Applications

Use the appropriate double-angle identity to simplify or solve. You may use:

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
$= 2\cos^2(\theta) - 1$
$= 1 - 2\sin^2(\theta)$

$\tan(2\theta) = (2\tan(\theta)) / (1 - \tan^2(\theta))$

1. Given $\sin(\theta) = 3/5$ and θ is in Quadrant II, find $\sin(2\theta)$
2. Given $\cos(\theta) = 4/5$ and θ is in Quadrant I, find $\cos(2\theta)$
3. Given $\tan(\theta) = 2$, find $\tan(2\theta)$
4. Simplify: $\sin(2x)$ if $\sin(x) = 1/2$ and $\cos(x) = \sqrt{3}/2$
5. Simplify: $\cos(2x)$ if $\sin(x) = 5/13$ and x is in Quadrant II
6. Simplify: $\tan(2x)$ if $\tan(x) = 1/\sqrt{3}$

✅ Half-Angle Formulas

🔹 **Sine Half-Angle:**

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

🔹 **Cosine Half-Angle:**

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

🔹 **Tangent Half-Angle:**

There are three equivalent forms:

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1+\cos(\theta)} = \frac{1-\cos(\theta)}{\sin(\theta)} = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$$

⚠️ Note on the ± Sign:

The ± depends on the **quadrant** where $\theta/2$ lies:

- Use **positive** or **negative** based on the **sign of the trig function** in that quadrant.

◆ **Apply Half-Angle Formulas**

Use half-angle identities to evaluate or simplify the expressions.

1. Use a half-angle formula to find sin(15°)
2. Use a half-angle formula to find cos(22.5°)
3. Use a half-angle formula to simplify $\sin^2(\theta/2)$ if $\cos(\theta) = 0.6$ and $0 < \theta < \pi/2$
4. Use a half-angle formula to simplify $\cos^2(\theta/2)$ if $\cos(\theta) = -0.8$ and $\pi < \theta < 2\pi$
5. Find an exact value for $\sin(\pi/8)$ using the half-angle formula

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Inverse Trig Functions & The Unit Circle

Part A: Evaluate the Following Without a Calculator

Use the unit circle to find exact values. Give answers in radians.

1. $\sin^{-1}\left(\frac{1}{2}\right) =$
2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$
3. $\tan^{-1}(1) =$
4. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$
5. $\cos^{-1}(0) =$
6. $\tan^{-1}(\sqrt{3}) =$
7. $\cos^{-1}(-1) =$
8. $\sin^{-1}(0) =$
9. $\tan^{-1}(0) =$
10. $\sin^{-1}\left(-\frac{1}{2}\right) =$